American Stock Option

American Option Introduction

- An American option give an investor the right but not the obligation to buy a call or sell a put at a set strike price at any time prior to the contract's expiry date.
- An investor holding an American-style option and seeking optimal value prefer to sell it on, rather than exercise it before maturity under certain circumstances.
- Investors and traders can use equity options to take a long or short position in a stock without actually buying or shorting the stock.
- Since investors have the freedom to exercise their American options at any point during the life of the contract, they are more valuable than European options, which can only be exercised at maturity.
- Investors/traders can therefore profit more from a price movement in the underlying stock.
- American options provide investors a way to hedge risk or speculate. Also option trading can limit an investor's risk and leverage investing potential.
- Option investors have a number of strategies they can utilize, depending on risk tolerance and expected return.
- Buying call options allows you to benefit from an upward price movement. The right to buy stock at a fixed price becomes more valuable as the price of the underlying stock increases.
- Put options may provide a more attractive method than shorting stock for profiting on stock price declines.
- If you have an established profitable long stock position, you can buy puts to protect this position against short-term stock price declines.
- An option seller earns the premium if the underlying stock price would not change much.
- This is advantageous because taking a position with options allows the investor/trader more leverage in that the amount of capital needed is much less than a similar outright long or short position on margin.

Valuation

- In general, American options do not have a closed-form solution.
- There are several methods to approximate the price of an American option: Roll-Geske-Whaley, Barone-Adesi and Whaley, Bjerksund and Stensland.
- To accreately value an American option, one needs to use a numerical approach.

- The most popular numerical methods are tree, lattice, PDE and Monte Carlo.
- FinPricing is using the Black-Scholes PDE plus finite difference method to price an American equity option.
- The finite difference model is one of the most widely used methods of approximation to solve the partial differential equation (PDE) equation for American options.
- There are three finite difference approximations most widely used for pricing American options: the Explicit, Fully Implicit and Crank-Nicolson models.

- Among these three, the Crank-Nicolson is unconditionally stable with respect to domain discretization and is the most accurate.

- Compared with the binomial tree method or the Whaley model, Crank-Nicolson also has obvious advantages of computation speed and accuracy.

- The Black-Scholes PDE for American option is:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} - rV \leq 0$$

where

S – the underlying stock price

V – the price of the American optin as a function of time and the underlying stock price

σ – the volatility of the underlying stock

r – the continuously compounded risk-free interest rate

q – the dividend yield

t – the time in years

- The boundary condition for an American call option is given by

$$C(S,T) = \max(S-K,0),\ C(0,t) = 0,\ C(S \to \infty, t) = S$$

- The boundary condition for an American put option is given by

$$P(S,T) = \max(K-S,0),\ P(0,t) = S,\ P(S \to \infty, t) = 0$$

where

T – the expiry time

K – the exercise price

C and P – the payoffs for the call and put options respectively

On this grid, the point (*i,j*) corresponds to time $i\Delta t$ and stock price $j\Delta Z$. Let the value of the option at (*i,j*) be denoted by $f_{i,j}$. Values for *N* and *M* are chosen based on the life of the option and the desired level of accuracy.

The Crank-Nicolson finite difference model replaces the space and time derivatives with finite differences centered at an imaginary time step at $\left(i+\frac{1}{2}\right)$. Therefore, we obtain the following formula:

Reference:
https://finpricing.com/lib/EqBarrier.html