Basket Option Valuation

A basket option is an option whose payoff depends on the value of a portfolio or basket of assets. In other words, the payoff of a basket option is a weighted sum different assets that have been grouped together in a basket. The assets in an equity basket option could be equity indices or individual equity.

A basket option can be used to hedge the risk exposure to or speculate the market move on the underlying stock basket. Because it involves just one transaction, a basket option often costs less than multiple single options.

The most important feature of a basket option is its ability to efficiently hedge risk on multiple assets at the same time. Rather than hedging each individual asset, the investor can manage risk for the basket or portfolio, in one transaction. The benefits of a single transaction can be great, especially when avoiding the costs associated with hedging each and every component of the basket or portfolio.

Buying a basket of shares is an obvious way to participate in the anticipated rapid appreciation of a sector of the stock market, without active management. An investor bullish on a sector but wanting downside protection may favor a call option on a basket of shares from that sector.

Trading desks use this type of option to construct the payoff structures in various Equity Linked Notes to issue. A trader who thinks the market overestimates a basket's volatility may sell a butterfly spread on the basket. A relatively risk adverse investor may favor a basket buy or write. A relatively risk adverse investor may favor a basket buy or write.

A trader who anticipates that the average correlation among different shares is going to increase might buy a basket option, hedge against a change in volatility by selling options on the component shares, and delta-hedge the remaining exposure to the underlying shares. This presentation gives an overview of equity basket option definition and valuation.

A basket offers a combination of two contradictory benefits: focus on an investment style or sector, and diversification across the spectrum of stocks in the sector.

In a basket option, the payoff is determined by the weighted average prices of the underlying stocks in a basket. This is different from the case of the usual European option and American option, where the payoff of the option contract depends on the price of the only one underlying instrument at exercise.

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The payoff for a basket call option is given by

$$Payoff_C(T) = N \cdot P \cdot \max(R - K, 0)$$

where

$R = \sum_{i=1}^{n} w_i S_i / F_i$ the weighted average of the basket return

N the notional amount

P the option participation rate

w_i the weight for asset i, $i = 1, \ldots, n$

F_i the InitialFixing for asset i, $i = 1, \ldots, n$

K the basket percentage strike

S_i the spot price for asset i at time T

The payoff for a basket put option is given by

$$Payoff_P(T) = N \cdot P \cdot \max(K - R, 0)$$

The basket option payoff function can be solved either analytically or using sample paths generated with the independent Monte Carlo engine.

Analytical solution assumes that the basket price can be approximated by a lognormal distribution with moments matched to the distribution of the weighted sum of the individual stock prices. The model includes two- and three-moment matching algorithms.

It is well known that the sum of a series of lognormal random variables is not a lognormal random variable. The weighted summation R is approximated by a shifted lognormal random variable (SLN).

$$R \sim SLN = c + d \cdot exp\left(\frac{Z-a}{b}\right)$$

where $Z \sim N(0,1)$ follows a standard normal distribution.

We can solve a, b, c, d by matching central moments below:

$$M_1 = c + d \cdot exp\left(\frac{1}{2b^2} - \frac{a}{b}\right)$$

$$M_2 = d^2 exp\left(\frac{1}{b^2} - \frac{2a}{b}\right)\left(exp\left(\frac{1}{b^2}\right) - 1\right)$$

$$M_3 = b^3 exp\left(\frac{3}{2b^2} - \frac{3a}{b}\right)\left(exp\left(\frac{1}{b^2}\right) - 1\right)^2 \left(exp\left(\frac{1}{b^2}\right) + 2\right)$$

The solved a, b, c, d are given by

$$b = \left[ln\left(\sqrt[3]{1 + \frac{\theta}{2} + \sqrt{\theta + \frac{\theta^2}{4}}} + \sqrt[3]{1 + \frac{\theta}{2} - \sqrt{\theta + \frac{\theta^2}{4}}} - 1\right)\right]^{-0.5}$$

$$\theta = \frac{M_3^2}{M_2^3}$$

$$d = sign\left(\frac{M_2}{M_3}\right)$$

$$a = b \cdot ln\left(\frac{M_2}{M_3} \cdot d \cdot exp\left(\frac{1}{2b^2}\right)\left(exp\left(\frac{1}{b^2}\right) - 1\right)\left(exp\left(\frac{1}{b^2}\right) + 2\right)\right)$$

$$c = M_1 - d \cdot exp\left(\frac{1}{2b^2} - \frac{a}{b}\right)$$

After some math, we get the present value of a call basket option as

$$PV_C = (c - K)\left(1 - \Phi\left(b \cdot ln\left(\frac{K-c}{d}\right) + a\right)\right)D$$
$$+ d \cdot exp\left(-\frac{a}{b} + \frac{1}{2b^2}\right)\left(1 - \Phi\left(b \cdot ln\left(\frac{K-c}{d}\right) + a - \frac{1}{b}\right)\right)D$$

where D is the discount factor.

Example

Face Value	87.5
Currency	USD
Digital Rebate	1
Maturity Date	6/16/2017
Call or Put	Call
Buy or Sell	Sell
Position	-21800
Underlying Assets	Initial Fixing
CTXS.O	87.5
LOGM.O	87.5

Reference:
https://finpricing.com/lib/FiBond.html