Spread Option Valuation

A *spread* option is an option written on the difference of two underling assets, whose values at time \( t \) we denote by \( S_1(t) \) and \( S_2(t) \). We consider only options of the European type for which the buyer has the right to be paid, at the maturity date \( T \), the difference \( S_2(T) - S_1(T) \), known as the spread. To exercise the option, the buyer must pay at maturity a prespecified price \( K \), known as the *strike*, or the *exercise price* of the option.

The use of spread options is widespread for speculation, basis risk mitigation, or even asset valuation. Spread options allow investors to simultaneously take positions in two are more assets and profit from their price difference over some spread.

Because of their generic nature, spread options are used in markets as varied as equity markets, fixed income markets, currency and foreign exchange markets, commodity futures markets, and energy markets.

Although a spread option is an option written on the difference between the values of two indexes in equity markets, its definition has been loosened to include all forms of options written as a linear combination of a finite set of indexes.

In the currency and fixed income markets, spread options are based on the difference between two interest or swap rates, two yields, etc. In the commodity markets, spread options are based on the differences between the prices of the same commodity at two different locations or between the prices of the same commodity at two different points in time, or between the prices of inputs to, and outputs from, a production process, as well as between the prices of different grades of the same commodity.

The spread can be calculated via one of two formulas: the *difference* of returns between two assets,

\[
Spread = \frac{S_1(T) - S_2(T)}{X_1 - X_2}
\]

or the *ratio* of returns of two assets:

\[
Spread = \frac{S_1(T)/X_1}{S_2(T)/X_2}
\]

Where \( X_i \) denotes the *Initial Level* of Asset \( i \).
A standard Call or Put payoff applies on the spread. The payoff may also be capped at some specified level:

\[
\text{Call Payoff} = \min\left[ \text{Cap}, \max\left[0, \text{Spread} - K \right]\right]
\]

\[
\text{Put Payoff} = \min\left[ \text{Cap}, \max\left[0, K - \text{Spread}\right]\right]
\]

Reference:
https://finpricing.com/lib/EqBarrier.html