Interest Rate Bermudan Swaption Valuation and Risk
Summary

- Bermudan Swaption Definition
- Bermudan Swaption Payoffs
- Valuation Model Selection Criteria
- LGM Model
- LGM Assumption
- LGM calibration
- Valuation Implementation
- A real world example
Bermudan Swaption

Bermudan Swaption Definition

- An interest rate Bermudan swaption is an option on an interest rate swap with predefined exercise schedules.
- A Bermudan swaption gives the holder the right but not the obligation to enter an interest rate swap at predefined dates.
- Bermudan swaptions give the holders some flexibility to enter swaps.
- A comparison of European, American and Bermudan swaptions
  - European swaption has only one exercise date at the maturity.
  - American swaption has multiple exercise dates (daily)
  - Bermudan swaption has multiple exercise dates (but not daily): such as quarterly, monthly, etc.
Bermudan Swaption Payoffs

- At the maturity $T$, the payoff of a Bermudan swaption is given by
  \[ Payoff(T) = \max(0, V_{swap}(T)) \]
  where $V_{swap}(T)$ is the value of the underlying swap at $T$.

- At any exercise date $T_i$, the payoff of the Bermudan swaption is given by
  \[ Payoff(T_i) = \max(V_{swap}(T_i), I(T_i)) \]
  where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.
Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.

The selection of interest rate term structure models

- Popular interest rate term structure models:
  - Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.
The selection of numeric approaches

After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.

Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.

Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.

Therefore, we choose either PDE or lattice.

Our decision is to use LGM plus lattice.
LGM Model

- The dynamics
  \[ dX(t) = \alpha(t)dW \]
  where \( X \) is the single state variable and \( W \) is the Wiener process.

- The numeraire is given by
  \[ N(t, X) = \left( H(t)X + 0.5H^2(t)\zeta(t) \right)/D(t) \]

- The zero coupon bond price is
  \[ B(t, X; T) = D(T)\exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right) \]
The LGM model is mathematically equivalent to the Hull-White model but offers:
- Significant improvement of stability and accuracy for calibration.
- Significant improvement of stability and accuracy for sensitivity calculation.

The state variable is normally distributed under the appropriate measure.

The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfectly correlated.
Bermudan Swaption

LGM calibration

- Match today’s curve
  
  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today’s discount curve.

- Select a group of market swaptions.

- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.
Bermudan Swaption

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the underlying swap value at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the Bermudan swaption.
### Bermudan Swaption

A real world example

<table>
<thead>
<tr>
<th>Swaption definition</th>
<th>Leg 1</th>
<th>Leg 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counterparty</td>
<td>xxx</td>
<td>xxx</td>
</tr>
<tr>
<td>Buy or sell</td>
<td>Sell</td>
<td></td>
</tr>
<tr>
<td>Payer or receiver</td>
<td>Receiver</td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td>USD</td>
<td></td>
</tr>
<tr>
<td>Settlement</td>
<td>Cash</td>
<td></td>
</tr>
<tr>
<td>Trade date</td>
<td>9/12/2012</td>
<td></td>
</tr>
<tr>
<td>Underlying swap definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Day Count</td>
<td>dcAct360</td>
<td>dcAct360</td>
</tr>
<tr>
<td>Leg Type</td>
<td>Fixed</td>
<td>Float</td>
</tr>
<tr>
<td>Notional</td>
<td>250000</td>
<td>250000</td>
</tr>
<tr>
<td>Payment Frequency</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pay Receive</td>
<td>Receive</td>
<td>Pay</td>
</tr>
<tr>
<td>Start Date</td>
<td>9/14/2012</td>
<td>9/14/2012</td>
</tr>
<tr>
<td>End Date</td>
<td>9/14/2022</td>
<td>9/14/2022</td>
</tr>
<tr>
<td>Fix rate</td>
<td>0.0398</td>
<td>NA</td>
</tr>
<tr>
<td>Index Type</td>
<td>NA</td>
<td>LIBOR</td>
</tr>
<tr>
<td>Index Tenor</td>
<td>NA</td>
<td>1M</td>
</tr>
<tr>
<td>Index Day Count</td>
<td>NA</td>
<td>dcAct360</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exercise Schedules</th>
<th>Exercise Type</th>
<th>Notification Date</th>
<th>Settlement Date</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call</td>
<td>1/12/2017</td>
<td>1/14/2017</td>
</tr>
<tr>
<td></td>
<td>Call</td>
<td>1/10/2018</td>
<td>1/14/2018</td>
</tr>
</tbody>
</table>
Thanks!

Reference: https://finpricing.com/lib/EqWarrant.html