Interest Rate Cancelable Swap Valuation and Risk
Cancelable Swap

Summary

- Cancelable Swap Definition
- Bermudan Swaption Payoffs
- Valuation Model Selection Criteria
- LGM Model
- LGM Assumption
- LGM calibration
- Valuation Implementation
- A real world example
A cancelable swap gives the holder the right but not the obligation to cancel the swap at predetermined dates prior to maturity.

It can be decomposed into a vanilla swap and a Bermudan swaption.

\[
P_{\text{PV Cancellable Payer Swap}} = P_{\text{PV Payer Swap}} - P_{\text{PV Receiver Bermudan Swaption}}
\]

\[
P_{\text{PV Cancellable Receiver Swap}} = P_{\text{PV Receiver Swap}} - P_{\text{PV Payer Bermudan Swaption}}
\]

A vanilla swap is well understood. Hence we focus on Bermudan swaption for the rest of this presentation.

A Bermudan swaption gives the holder the right but not the obligation to enter an interest rate swap at predefined dates.
Bermudan Swaption Payoffs

- At the maturity $T$, the payoff of a Bermudan swaption is given by
  \[ Payoff(T) = \max(0, V_{swap}(T)) \]
  where $V_{swap}(T)$ is the value of the underlying swap at $T$.

- At any exercise date $T_i$, the payoff of the Bermudan swaption is given by
  \[ Payoff(T_i) = \max(V_{swap}(T_i), I(T_i)) \]
  where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.
Model Selection Criteria

Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.

The selection of interest rate term structure models

- Popular interest rate term structure models:
  - Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
- HJM and LMM are too complex.
- Hull-White is inaccurate for computing sensitivities.
- Therefore, we choose either LGM or QGM.
The selection of numeric approaches

After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.

Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.

Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.

Therefore, we choose either PDE or lattice.

Our decision is to use LGM plus lattice.
The dynamics

\[ dX(t) = \alpha(t)dW \]

where \( X \) is the single state variable and \( W \) is the Wiener process.

The numeraire is given by

\[ N(t, X) = \frac{H(t)X + 0.5H^2(t)\zeta(t)}{D(t)} \]

The zero coupon bond price is

\[ B(t, X; T) = D(T)\exp(-H(t)X - 0.5H^2(t)\zeta(t)) \]
LGM Assumption

◆ The LGM model is mathematically equivalent to the Hull-White model but offers
  ◆ Significant improvement of stability and accuracy for calibration.
  ◆ Significant improvement of stability and accuracy for sensitivity calculation.
◆ The state variable is normally distributed under the appropriate measure.
◆ The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.
LGM calibration

- Match today’s curve
  At time $t=0$, $X(0)=0$ and $H(0)=0$. Thus $Z(0,0;T)=D(T)$. In other words, the LGM automatically fits today’s discount curve.
- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.
Cancelable Swap

Valuation Implementation

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the underlying swap value at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date and also Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the Bermudan swaption.
- The final value of the cancelable swap is given by
  \[ PV_{\text{CancellablePayerSwap}} = PV_{\text{PayerSwap}} - PV_{\text{ReceiverBermudanSwaption}} \]
  \[ PV_{\text{CancellableReceiverSwap}} = PV_{\text{ReceiverSwap}} - PV_{\text{PayerBermudanSwaption}} \]
### Cancelable Swap

#### A real world example

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<td>Payer or receiver</td>
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<td>Settlement</td>
<td>Physical</td>
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#### Underlying swap definition

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#### Exercise Schedules

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<tr>
<td>Call</td>
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Thanks!

You can find more details at https://finpricing.com/lib/EqConvertible.html